

Moment Lyapunov exponents for a suspension bridge parametrically excited by large-scale turbulence

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SUMMARY:

This work addresses the stochastic stability of a simplified 2D suspension bridge model immersed in a turbulent flow. The variation of the angle of attack due to large-scale turbulence can parametrically excite the bridge, possibly leading to a reduction of the flutter stability limit. Rare sudden increases in the bridge response can also occur in some cases. The moment Lyapunov exponents are numerically calculated to deal with this problem, evaluating the system p -th stability.

Keywords: long-span bridges, stochastic stability, moment Lyapunov exponents

1. INTRODUCTION

At the dawn of a new era of super-long suspension bridges launched by the latest opening of the new Çanakkale Bridge, there is the perception that new challenges for wind engineers may appear for such an unexplored structural flexibility. Indeed, the buffeting response to turbulent wind may govern the structural design for these cutting-edge constructions. This means that the uncertainty associated with the aerodynamic load must be handled with care, as it significantly affects the overall structural reliability. In this regard, the nonlinear effects of turbulence in bridge buffeting response and flutter stability are still open issues, and the available experimental and numerical results revealed either a stabilising or a destabilising role (e.g., Bartoli and Spinelli, 1993; Billah and Shinozuka 1994; Huston 1986; Tsiatis and Gasparini 1987).

In the wake of Barni et al. (2022a), this work tries to understand better the nonlinear parametric excitation induced by the slow variation of the self-excited forces due to the angle of attack associated with large-scale atmospheric turbulence. These effects can induce rare but potentially catastrophic bridge oscillations even for a mean wind velocity significantly lower than the deterministic flutter stability threshold. The so-called 2D rational function approximation (RFA) model (Barni et al., 2021) is used for self-excited forces, leading to a stochastic model of the bridge. After simplifying the bridge structure to a three-degree-of-freedom 2D model, Moment Lyapunov Exponents (MLE) are calculated to evaluate the bridge sample and P-stochastic stability. Moment Lyapunov exponents are the most important indices to describe stochastic stability and bifurcation of a system. They can indicate that, although the response of an autonomous linear system decays to zero (with probability one) at a certain exponential rate, there is a small probability that the bridge response is large.

2. MATHEMATICAL MODEL

According to the 2D RFA model, when aerodynamic derivatives are sensitive to a low-frequency variation of the angle of attack, the transfer function between self-excited forces and bridge motion components maintains the same form valid for a linear system, but it becomes a function of the slowly-varying angle of attack $\tilde{\alpha}$ (in addition to the reduced frequency of oscillation). The variation of the angle of attack can easily be induced by large-scale turbulence, and since the random wind field varies in space and time, this angle of attack generally changes along the girder. Nevertheless, this important effect is not considered here for a 2D bridge model. Please refer to Barni et al. (2021) for the time-variant self-excited force equations.

According to the nonlinear approach for buffeting response presented in Barni et al. (2022a), the equation of motion can be written in a matrix form as follows:

$$\ddot{\mathbf{r}} = -\mathbf{M}^{-1}(\mathbf{C} + \mathbf{C}_{ae}(\tilde{\alpha}))\dot{\mathbf{r}} - \mathbf{M}^{-1}(\mathbf{K} + \mathbf{K}_{ae}(\tilde{\alpha}))\mathbf{r} + \frac{1}{2}\rho V_m^2 \mathbf{M}^{-1} \sum_{l=1}^{N-2} \mathbf{A}_{l+2}(\tilde{\alpha}) \boldsymbol{\Psi}_l + \mathbf{q}_{ext} \quad (1)$$

\mathbf{M} , \mathbf{C} and \mathbf{K} represent the structural mass, damping and stiffness matrices, respectively; while \mathbf{C}_{ae} , \mathbf{K}_{ae} are the aerodynamic damping and stiffness matrices directly deriving from the RFA approximation (see Barni et al., 2021), $\mathbf{r} = [y \ z \ \theta]^T$ is the bridge girder motion vector and $\boldsymbol{\Psi}_l$ are the $N - 2$ additional aeroelastic states. The external load vector \mathbf{q}_{ext} can also be obtained through a dynamic linearisation around the slowly-varying angle of attack, as explained in Barni et al. (2022a). Eq. (1) is a second-order stochastic differential equation, the parameters of which depend on $\tilde{\alpha}$, which is a stochastic process. Therefore, applying a state-space transformation through $\boldsymbol{\gamma}_1 = \mathbf{r}$, $\boldsymbol{\gamma}_2 = \dot{\mathbf{r}}$ and $\boldsymbol{\gamma}_{l+2} = \boldsymbol{\Psi}_l$, after some manipulation, the following stochastic differential equation is obtained:

$$\dot{\boldsymbol{\gamma}}(t) = \boldsymbol{\Omega}(\tilde{\alpha})\boldsymbol{\gamma}(t) + \mathbf{B}\mathbf{q}_{ext} \quad (2)$$

$\boldsymbol{\gamma} \in \mathbb{R}^{3(2+(N-2))}$ is the state vector, $\boldsymbol{\Omega}(\tilde{\alpha}) \in \mathbb{R}^{[3(2+(N-2))] \times [3(2+(N-2))]}$ is the time-variant state matrix of the system, and $\mathbf{B} = [\mathbf{0} \ \mathbf{M}^{-1} \ \mathbf{0}]^T \in \mathbb{R}^{[3(2+(N-2))] \times 3}$ is the input matrix.

According to Arnold et al. (1984, 1986), the p -th MLE of the homogeneous system associated with Eq. (2) is defined by:

$$\Lambda(p) = \lim_{t \rightarrow \infty} \frac{1}{t} \log\{E[\|\boldsymbol{\gamma}(t)\|^p]\} \quad (3)$$

where $E[\cdot]$ denotes the ensemble average and $\|\cdot\|$ the Euclidean norm. The p -th moment of the solution of the system is asymptotically stable if $\Lambda(p) < 0$. Arnold et al. (1984) showed that the slope in the origin $\Lambda(p = 0)$ of the moment Lyapunov exponent curve is equal to the largest Lyapunov exponent λ , commonly used to describe the system almost-sure or sample stability.

The MLE assessment can be important for engineering problems. Indeed, given an almost surely stable system (on average, the response decays to zero at an exponential rate λ), the process may still exceed some threshold values along the decay. This small probability of a large response makes the expected value of this rare event large for a large value of p , leading to an unstable p -th mean response.

3. NUMERICAL CALCULATION OF MOMENT LYAPUNOV EXPONENTS

In this work, MLEs are numerically obtained through Monte Carlo simulations. Once S sample solutions of the stochastic differential Eq. (2) have been obtained for a time $t_k = k\Delta t$, the p -th statistical moment can be determined as follows:

$$E\|\mathbf{Y}(t_k)\|^p = \frac{1}{S} \sum_{s=1}^S \left\| (\mathbf{Y}^{(k)})_s \right\|^p, \quad \left\| (\mathbf{Y}^{(k)})_s \right\| = \sqrt{(\mathbf{Y}^{(k)})_s^T (\mathbf{Y}^{(k)})_s} \quad (4)$$

Then, the MLE $\Lambda(p)$ can be determined based on Eq. (3). The algorithm is validated with a first-order stochastic differential equation reported in Xie and Huang (2009), for which the analytical expression of the MLEs is known. As shown in Fig. 1, by considering 10^6 samples of 10 s, with a time step of 0.01 s, the Monte Carlo estimation of MLEs is very accurate.

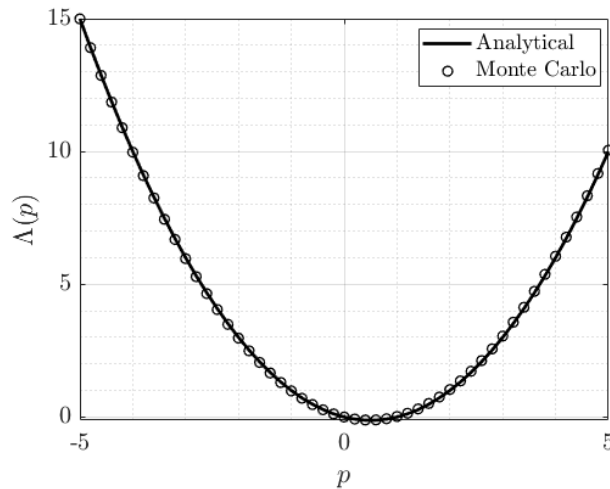


Figure 1. Calculation of moment Lyapunov exponents for the chosen validation test case.

4. STOCHASTIC STABILITY ANALYSIS AND CONCLUSIONS

The Monte Carlo approach is then used to determine the MLEs of Eq. (2), which governs the stochastic flutter stability of a bridge in turbulent flow. A 2D model of the Hardanger Bridge (three degrees of freedom) is considered, whose mechanical properties are reported in Barni et al. (2022b). Longitudinal turbulence intensity and integral length scale of 15% and 200 m, respectively, are set to generate the turbulent wind field. A mean wind velocity inclination of 2.5 deg is also considered, consistently with observations in the Hardanger Bridge site. 30-minute time histories of turbulent wind velocity components are generated with the method of Shinozuka and Ian, with a sampling frequency of 200 Hz.

Fig. 2(a) shows the lateral, vertical and torsional components of the bridge response for the unit hypersphere initial condition $\|\mathbf{Y}(t)\| = 1$ (top row) and due to the external buffeting force vector \mathbf{q}_{ext} (bottom row). In general, the increase in the response for the linear time-variant (LTV) self-excited force model compared to the standard invariant model (LTI) can be ascribed to the variation in the aerodynamic derivative A_2^* due to the slowly-varying angle of attack $\tilde{\alpha}$. Indeed, A_2^* assumes positive values (corresponding to negative aerodynamic damping in torsion) for a nose-up angle of attack higher than about 5 deg (Barni et al., 2021): Clearly, even if the response of the autonomous system decays to zero, the remarkable increase in the response due to

temporarily low or negative damping cannot be ignored. As shown in Fig. 2(b), this behaviour denotes a p -th moment instability successfully detected by moment Lyapunov exponents. Indeed, the parametric excitation due to large-scale turbulence can destabilise the statistical moments of order higher than two.

In this work, moment Lyapunov exponents have proven to be an effective mathematical tool for stochastic stability analysis, leading to a formal definition of bridge flutter stability threshold in turbulent flow. However, further work is still necessary for a better understanding of MLE convergence with respect to important parameters, such as the number and length of samples, and time resolution.

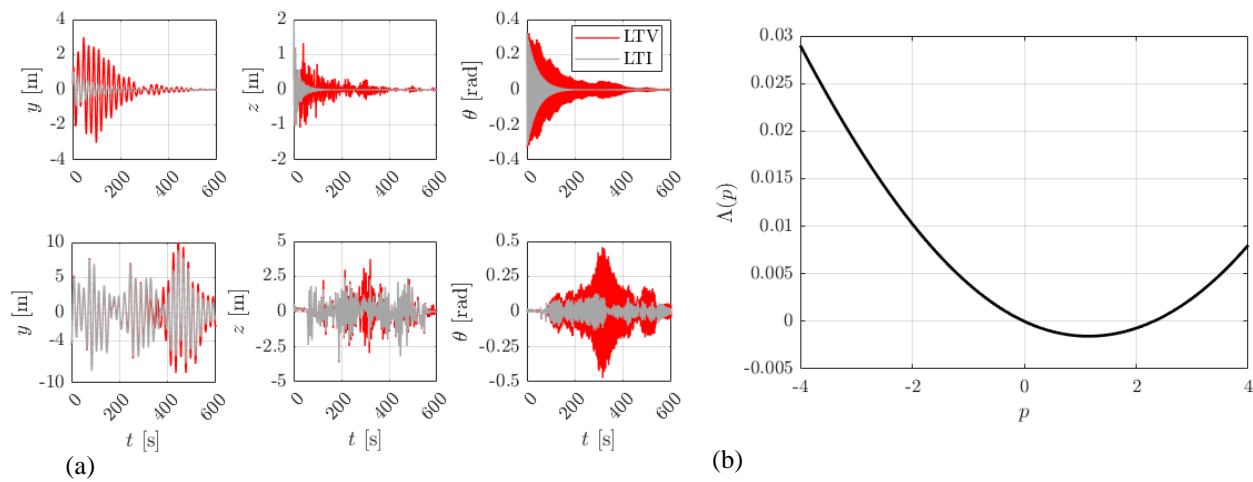


Figure 2. (a) Lateral, vertical and torsional components of the bridge response for the initial condition $\|\boldsymbol{\gamma}(t)\| = 1$ (top row) and due to an external buffeting load (bottom row); (b) moment Lyapunov exponents.

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